Global Positioning System (GPS)

2011

MIEET 1º ano

UAlg
UNIVERSIDADE DO ALGARVE
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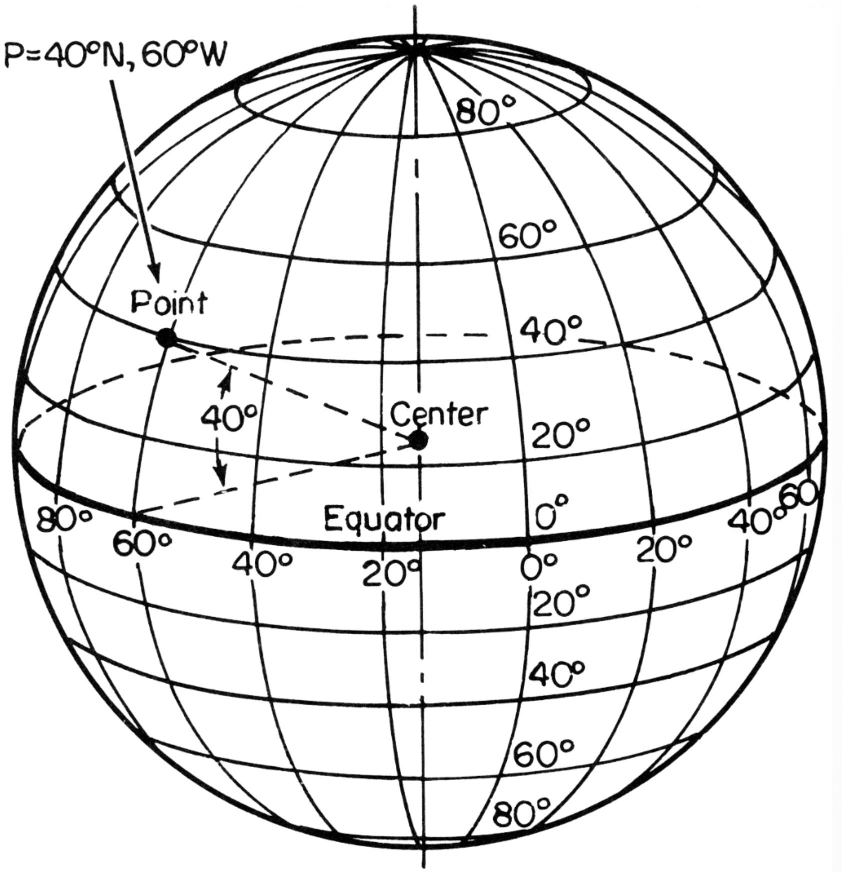
Why knowing position?

Knowing your position is needed for when you want to go somewhere (and don't get lost)

This is a seemingly simple problem that is (surprisingly) difficult!

Global coordinates



Position consists of:

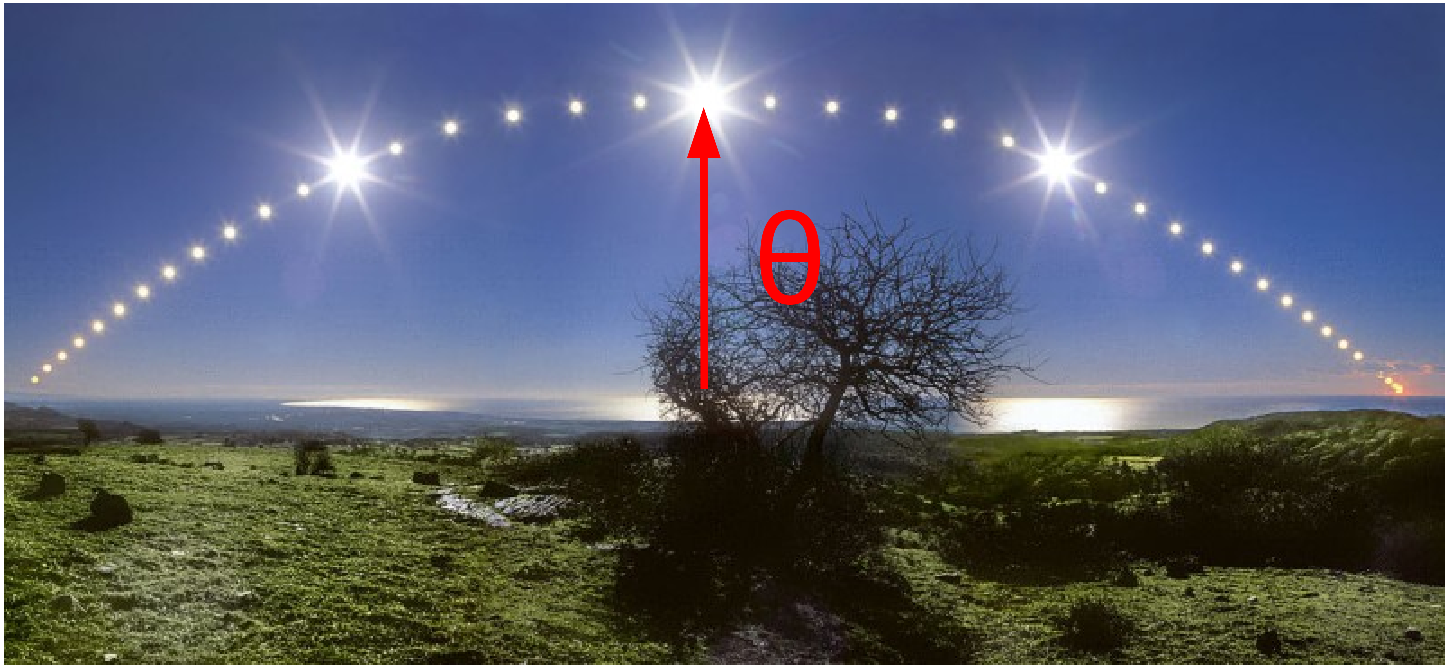
- **Longitude** (East-West)
  0^o is Greenwich (London)
- **Latitude** (North-South)
  0^o is equator, 90^o is N/S pole

Example: Faro (airport)
37°00′52″N 007°57′57″W

Latitude

- Latitude (North-South)

can be determined by height (inclination, θ) of sun in sky (at its highest point!)



Northern hemisphere (ex. Portugal): left to right →
Southern hemisphere (ex. Australia): right to left ←

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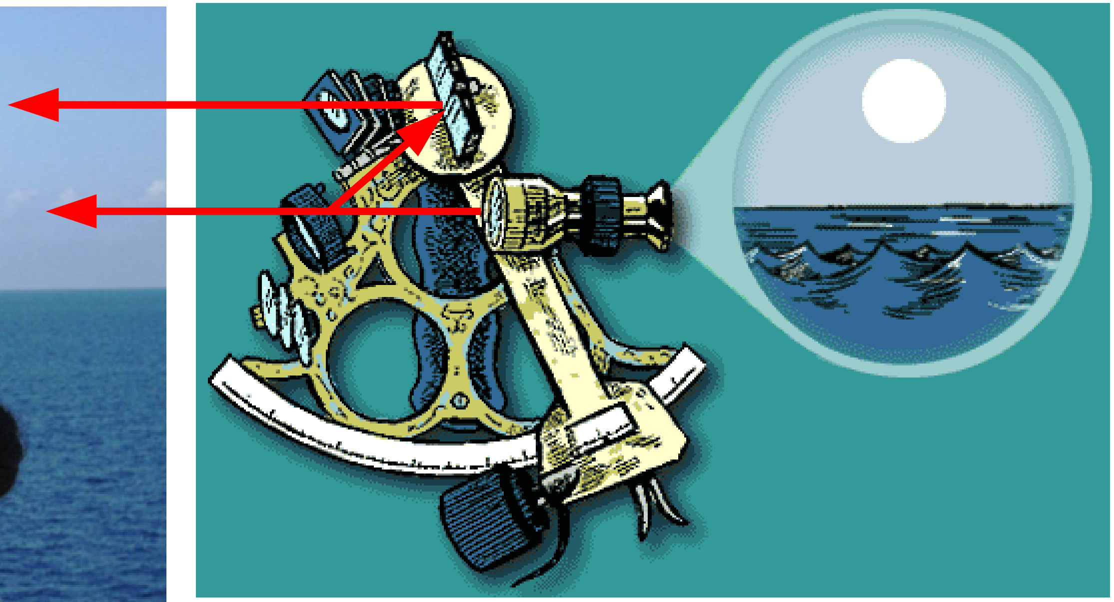
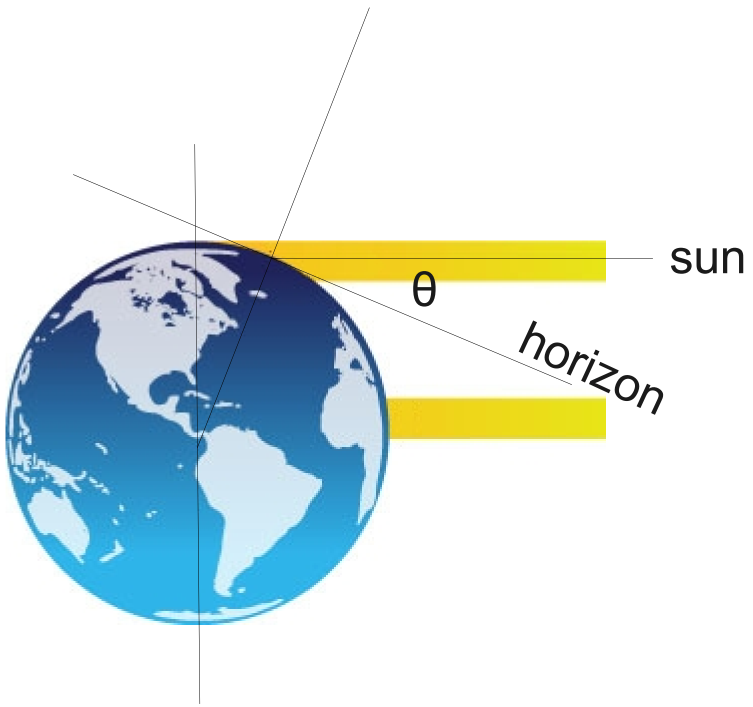


When sun is aligned with horizon, on the scale we can read the inclination

Latitude

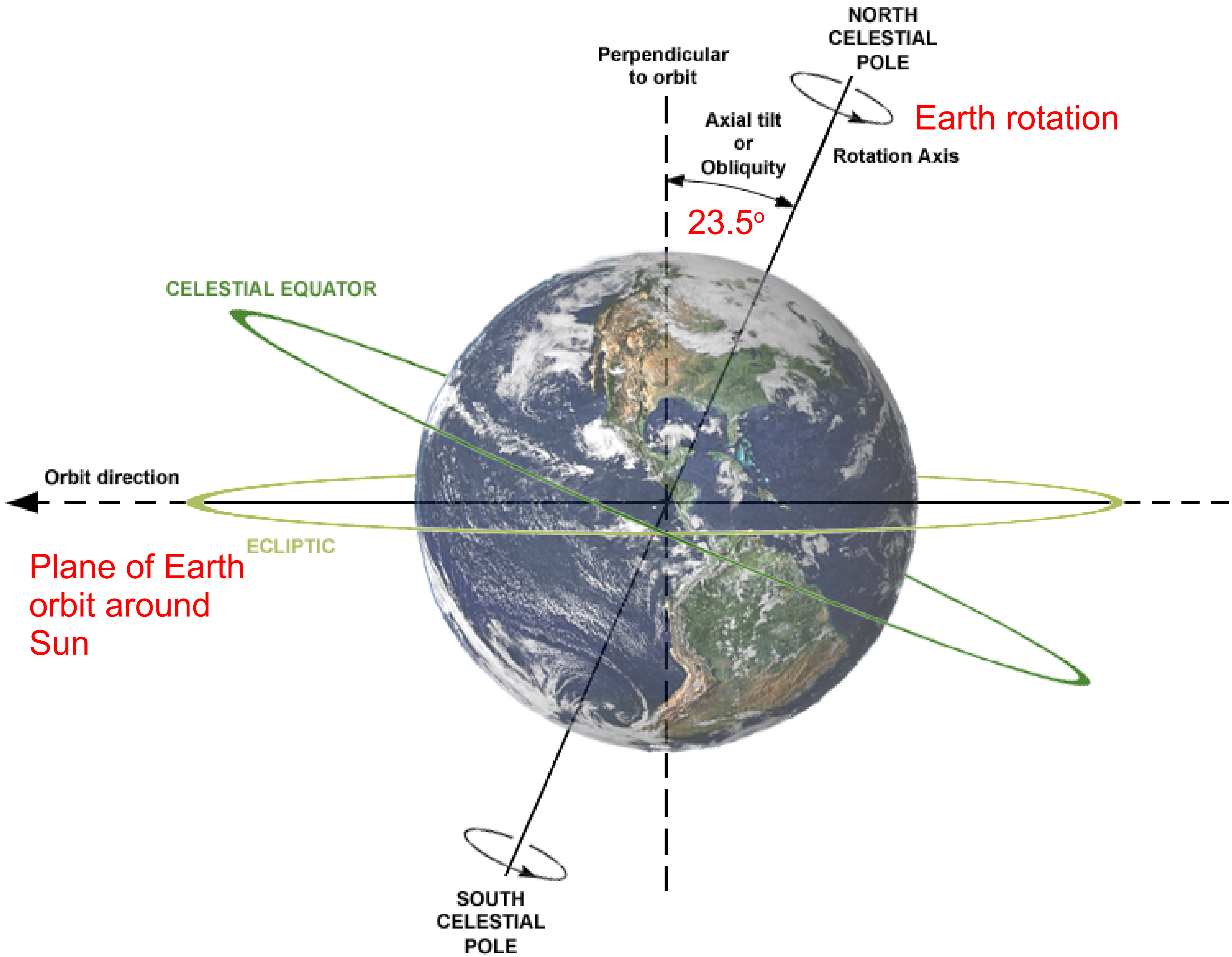
- Latitude (North-South)

can be determined by height (inclination, θ) of sun in sky (at its highest point! at midday)



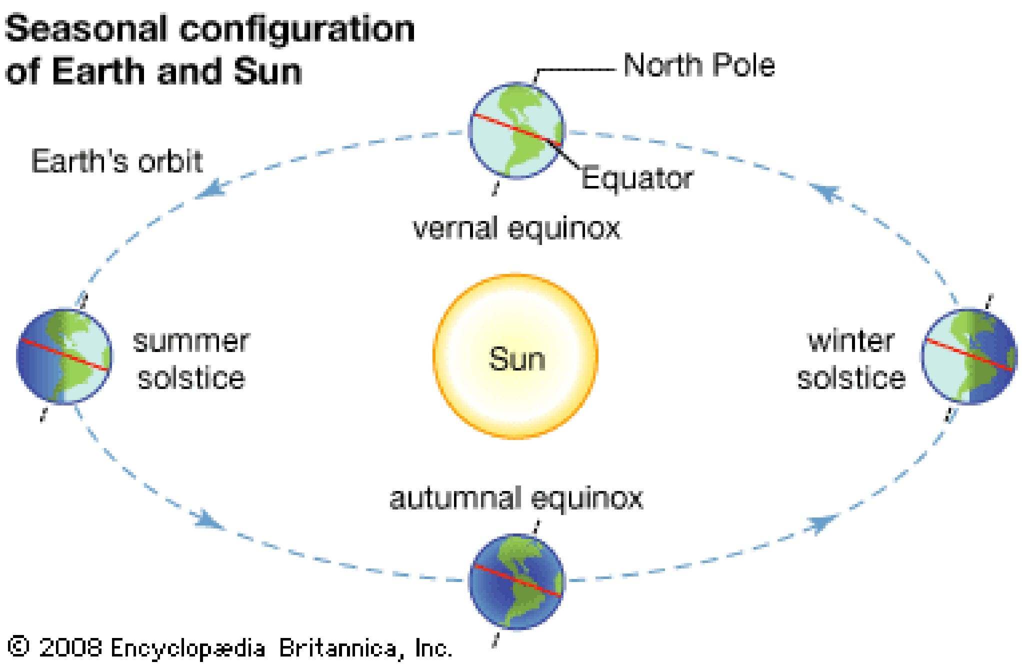
Simple idea: Latitude = $90^o - \theta$?
More complicated than that!

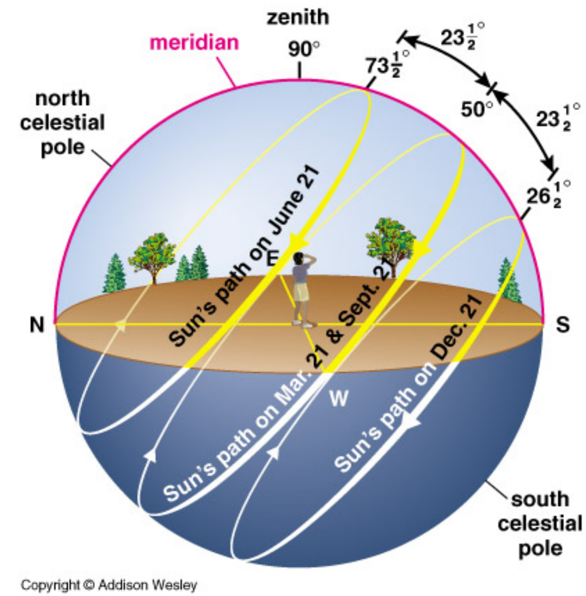
Latitude; Earth rotation axis

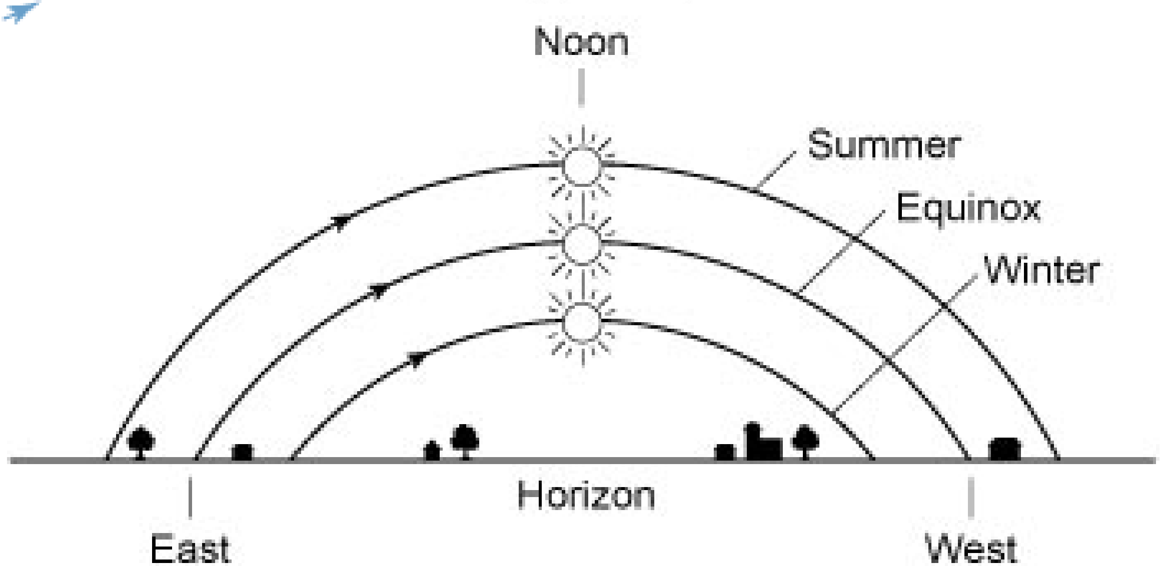


Latitude

The sun's inclination depends not only on time of day, but also on the day of the year.



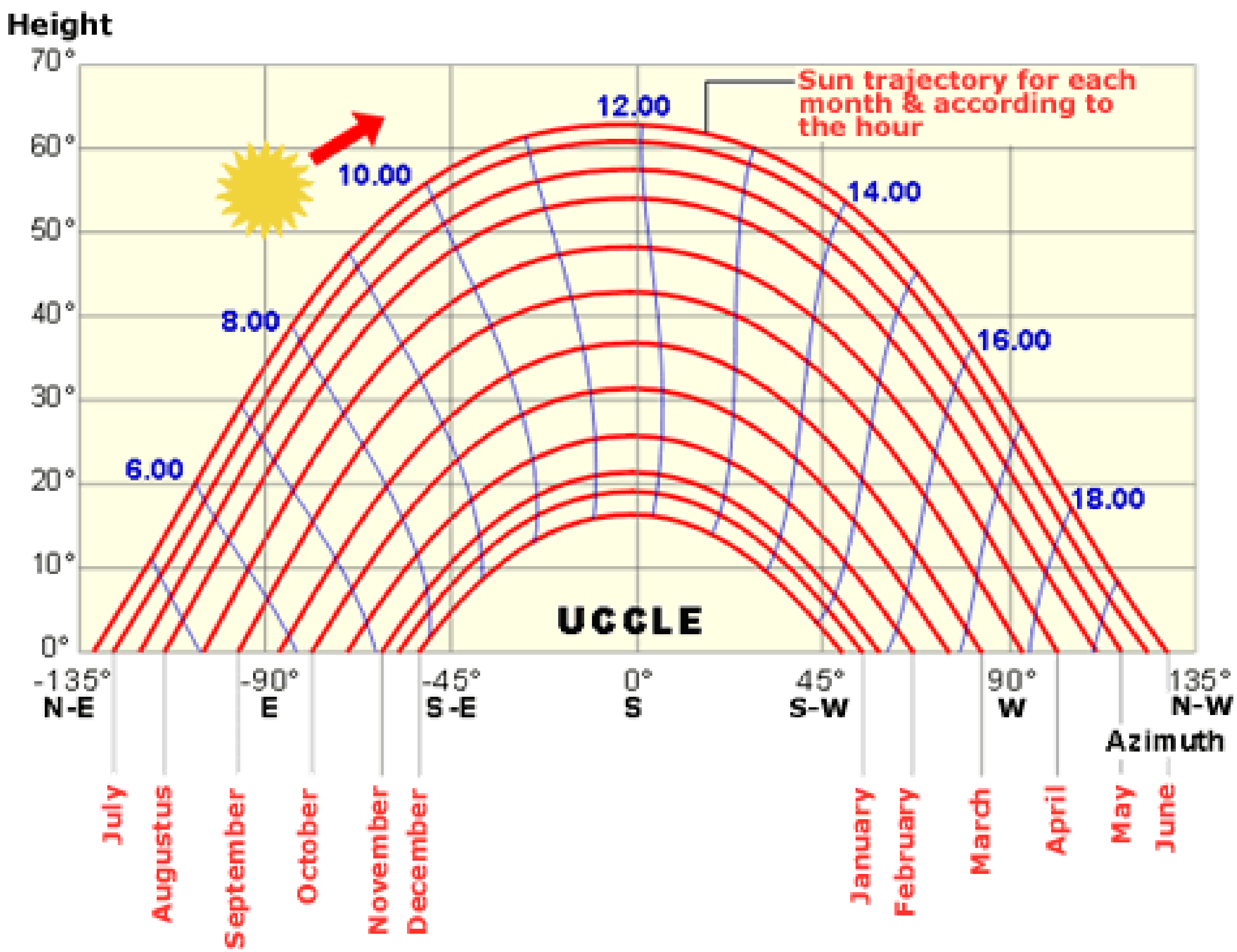




Latitude

- Latitude (North-South)
Very complicated indeed.

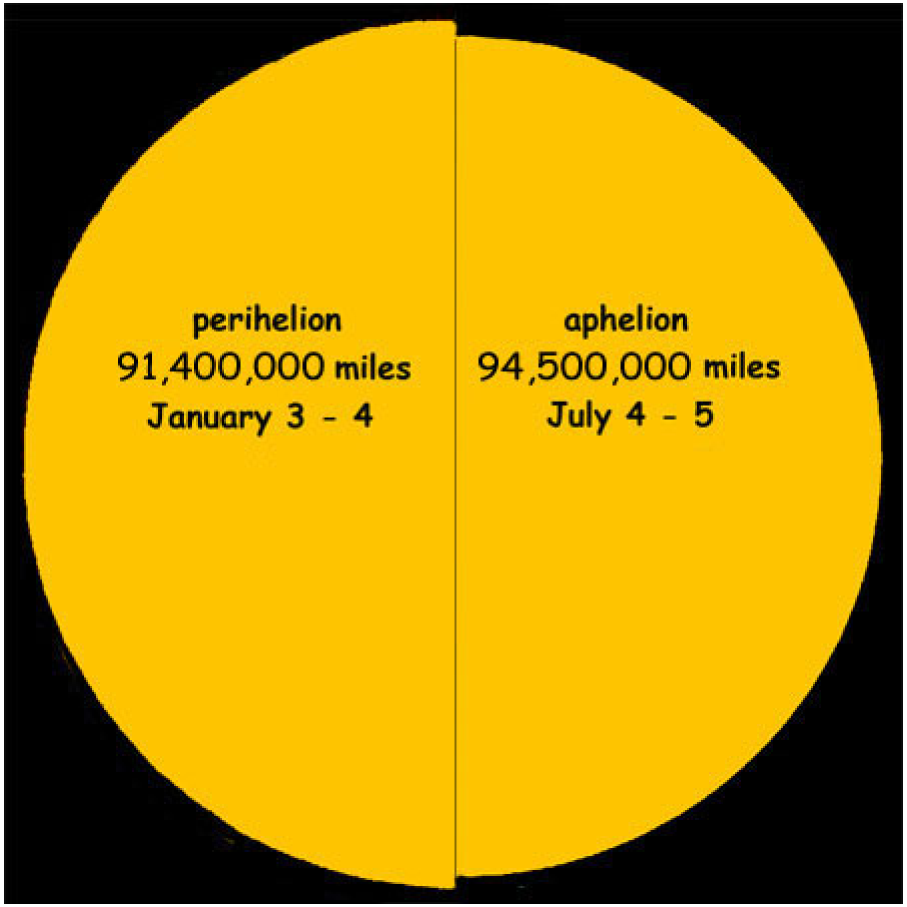
- Sun at highest point **not South**
- Sun **not** always at **same time** of day at highest point



Latitude

Position of sun at 12:00 along the year
(Analemma)

Latitude



The sun is not even the same size along the year.
Very difficult/unreliable object to use in positioning (and navigation)

Latitude

- Latitude (North-South)

Better use distant stars. They don't suffer from seasonal effects.

They still change during a day (trajectory).

Except one!

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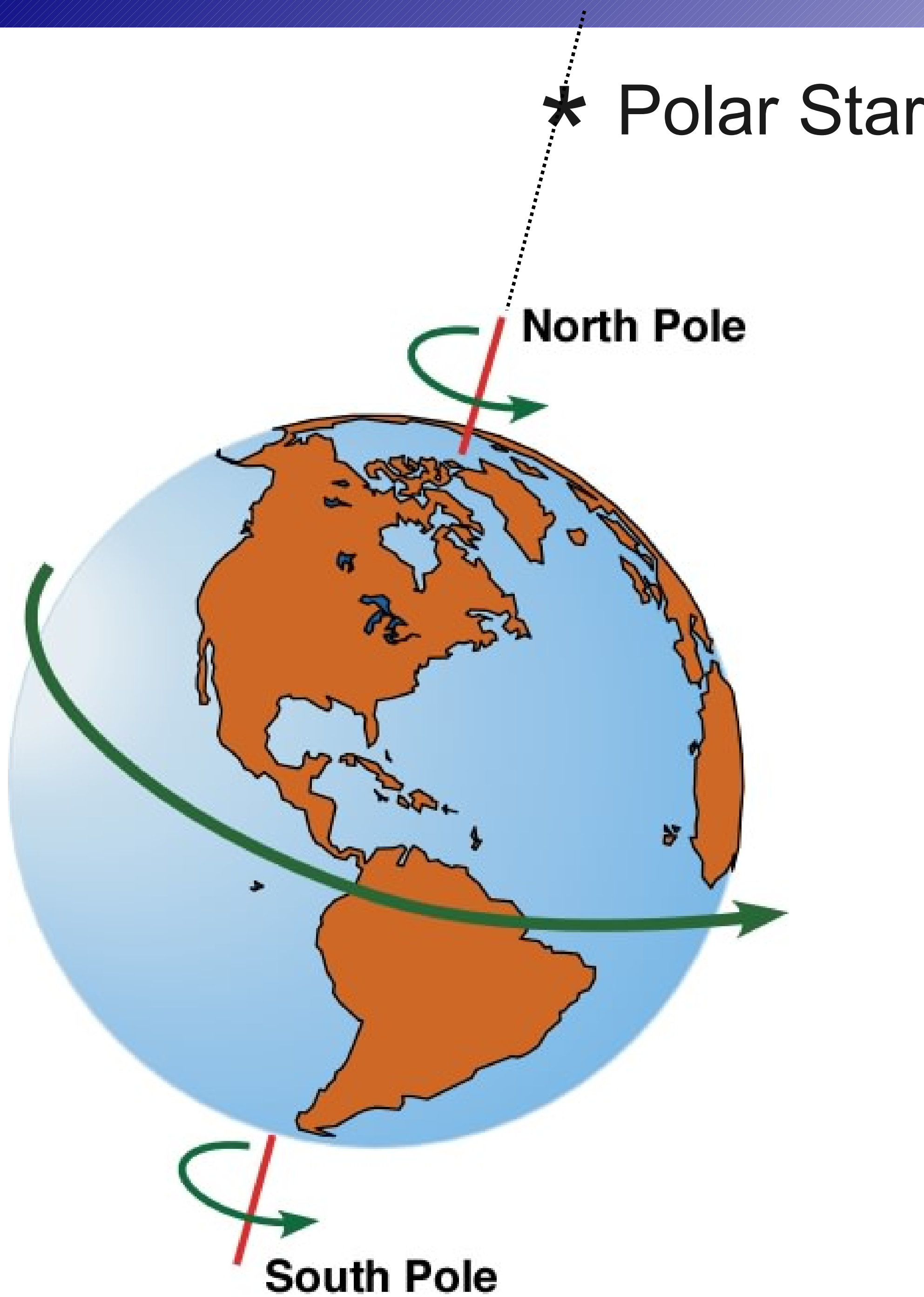
Except one!

The Polar Star

Long-time exposure at night

Latitude: Polar Star

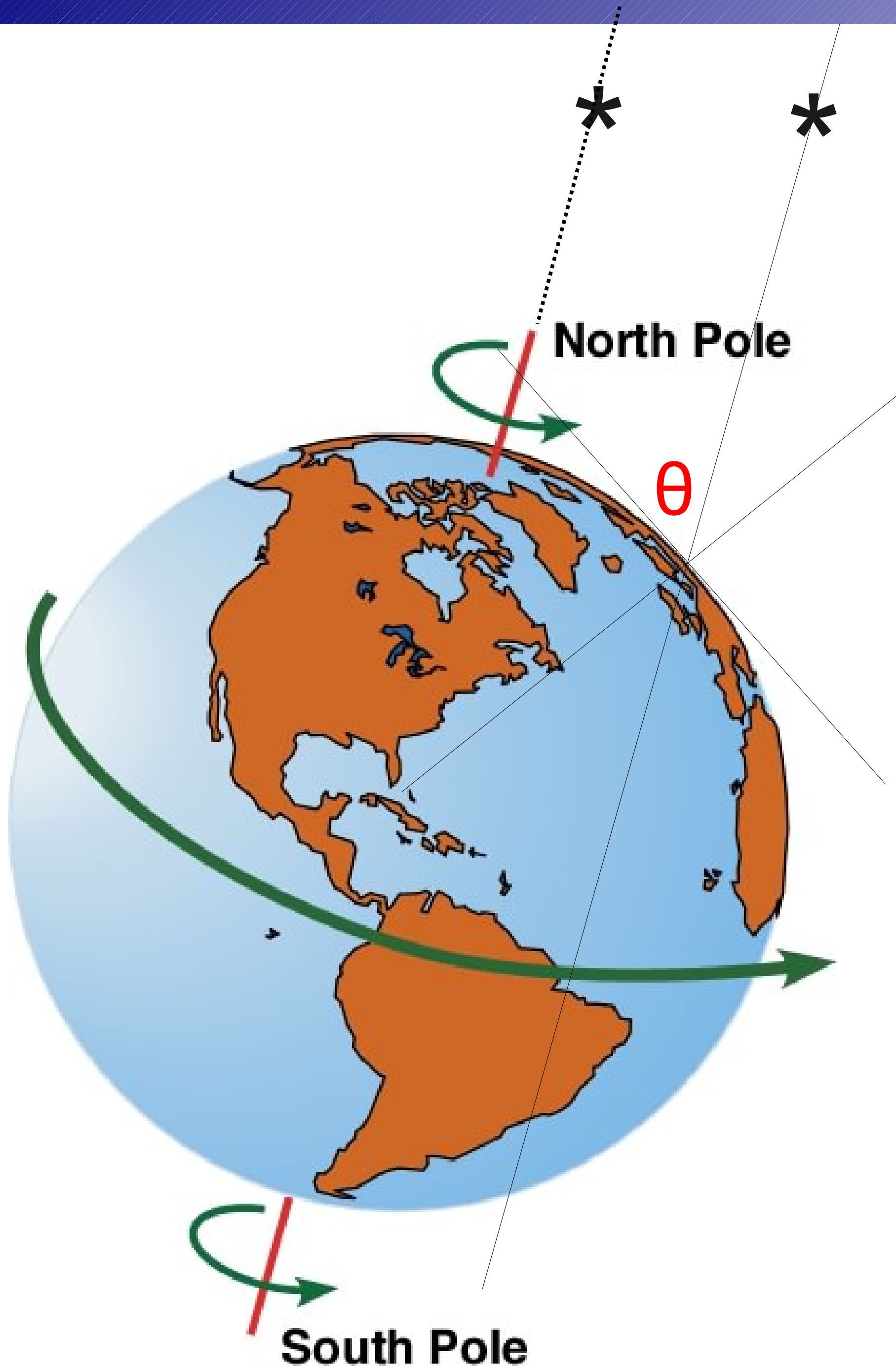


The Polar Star lies on the axis of the rotation of the Earth.
All day and all year at the same position in the sky.

Always at geographic North

Inclination equal to latitude

Latitude: Polar Star



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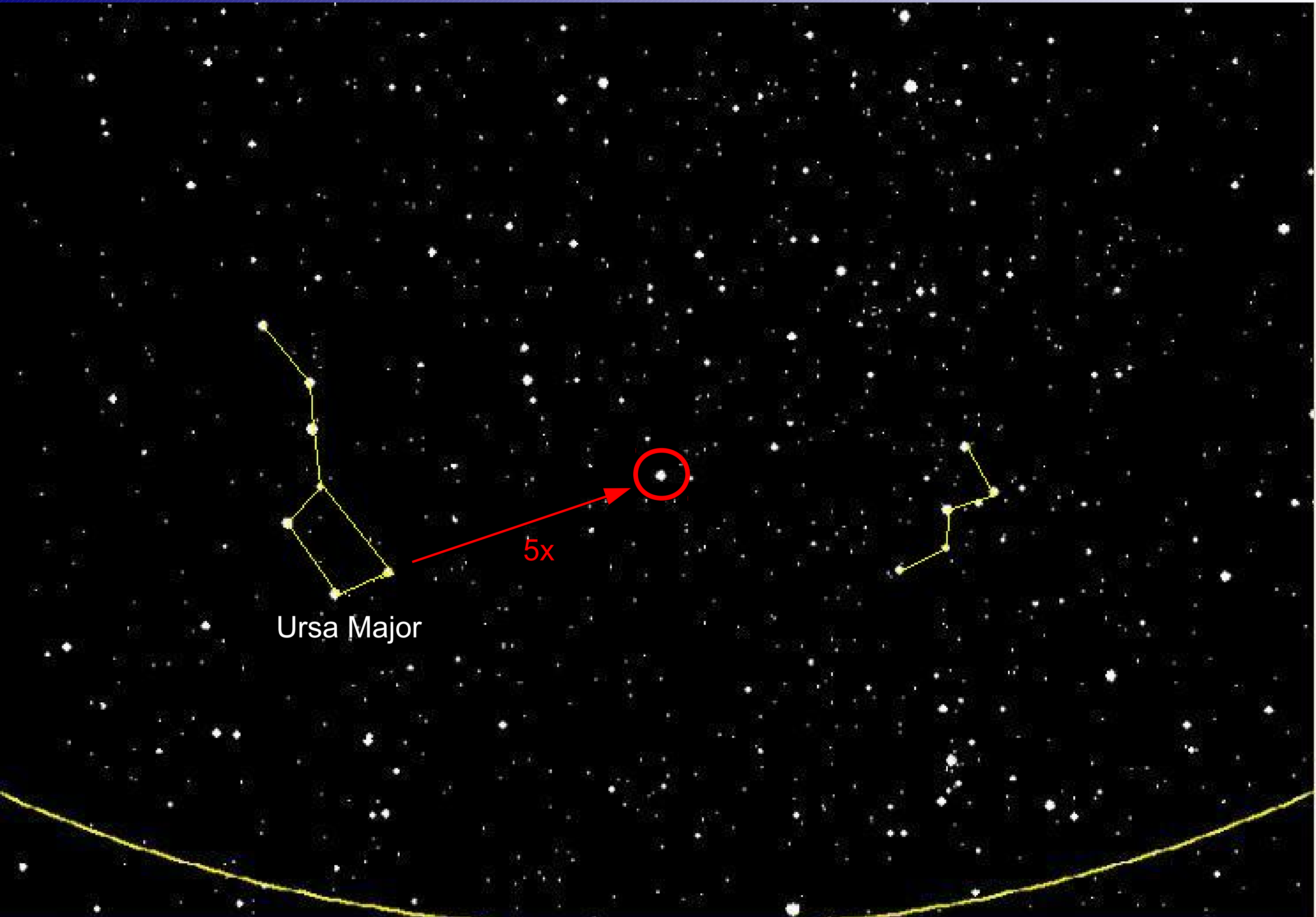
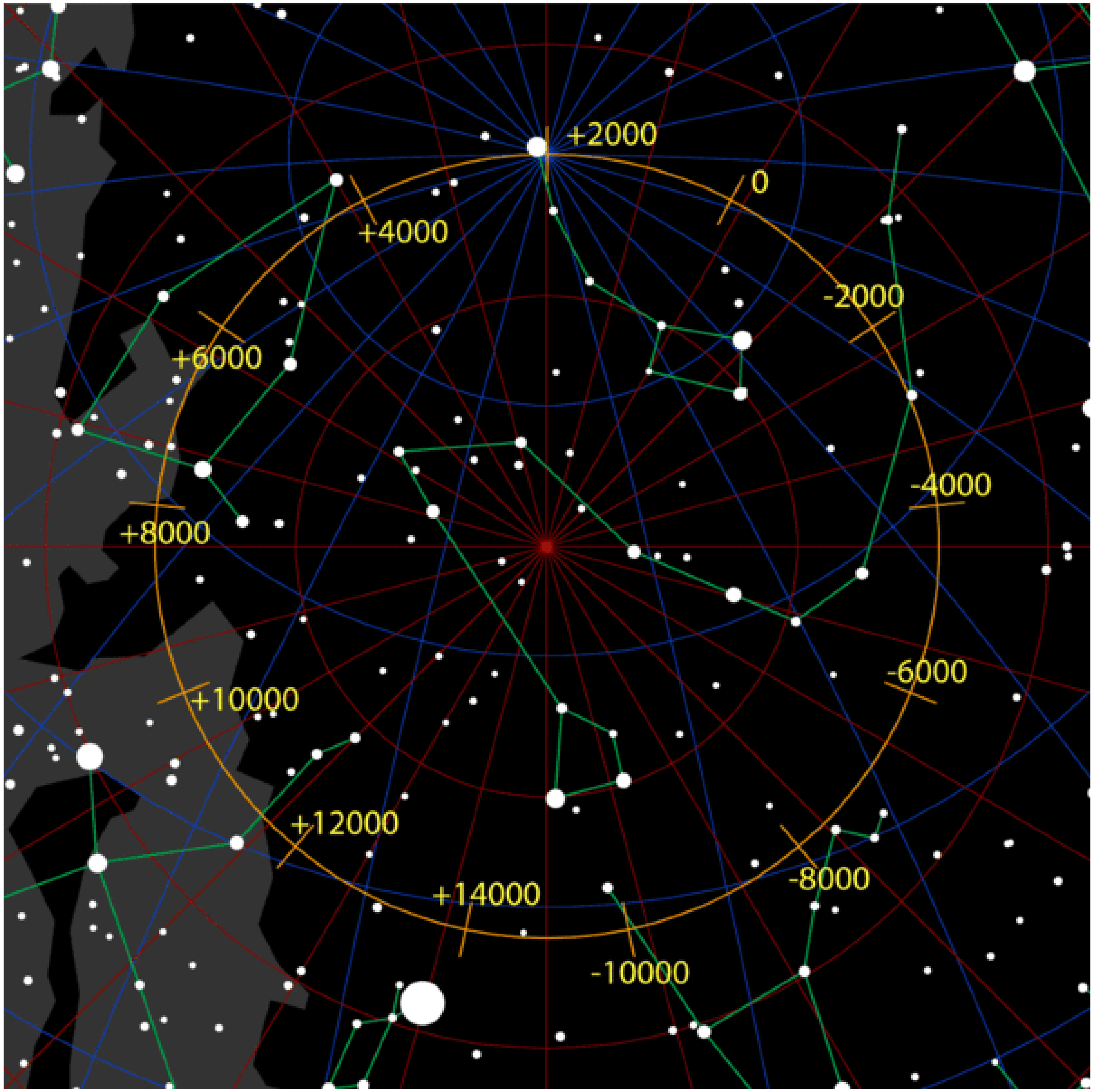


Latitude: Polar Star

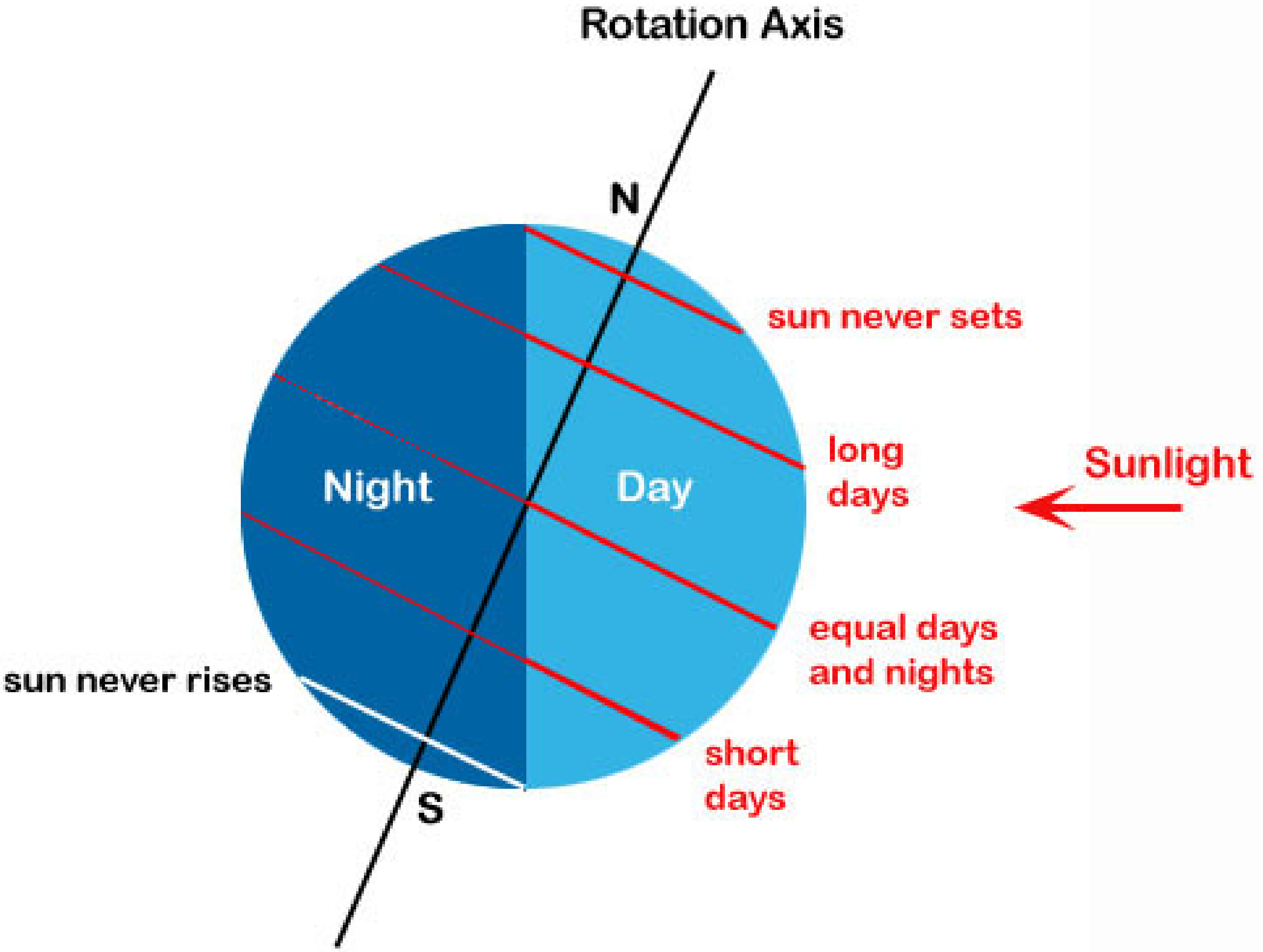


Not even Pole Star position is fixed!

Precession of Earth

Latitude: Polar Star

So far we have just found only the latitude.
The **easiest** part!



Longitude

Longitude? Time!

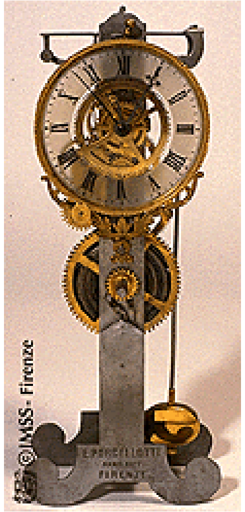
In Greenwich, the sun is at its highest point at exactly 12:00

West: 1 hour later for every 15^o (360^o/24 h)
East: 1 hour earlier for every 15^o

If we know the **time** (GMT) when the sun is at its highest, we know the **longitude**!

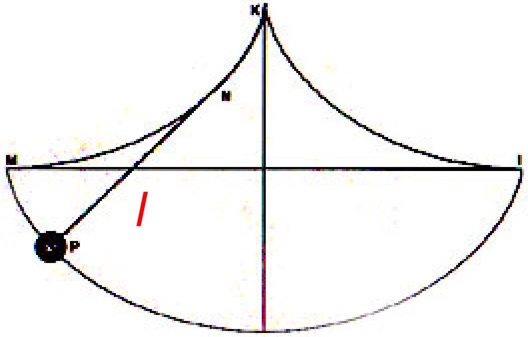
The need for precise clocks!

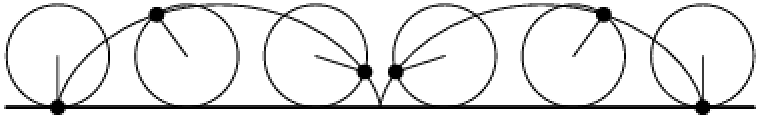
Longitude: Pendulum clock



Period (T=1/f) of a pendulum:

$$T = 2\pi\sqrt{\frac{l}{g}}$$





Period is independent of amplitude of swing if the curve is a cycloid (Christiaan Huijgens, 17th century)

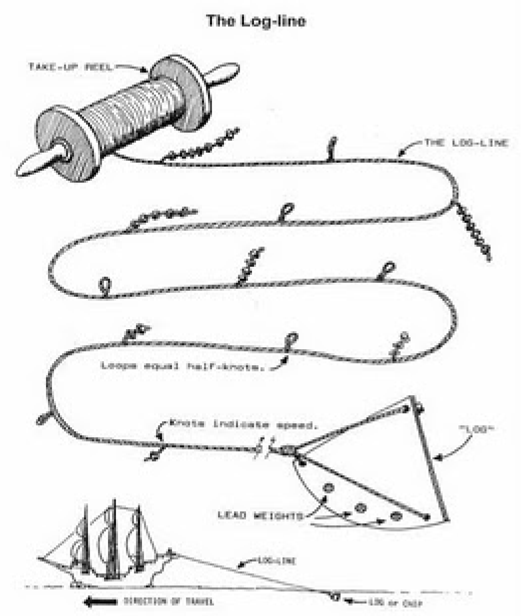
Comments

With a magnetic compass we cannot measure position, only direction

With a mechanical compass we can plot distance on a map (distance is speed x time)

Speed was measured with a rope with knots
measure how many equally spaced knots (47 feet, 3 inch) are dragged into the water in 30 seconds

1 knot is equivalent to one nautical mile per hour
1.852 km/h



GPS v. 1.0

So far we are still in the preindustrial era, but we know more-or less our position on this globe.

… and on the Southern Hemisphere? (no North Pole Star!)

… and what if it is cloudy?!!!

GPS.

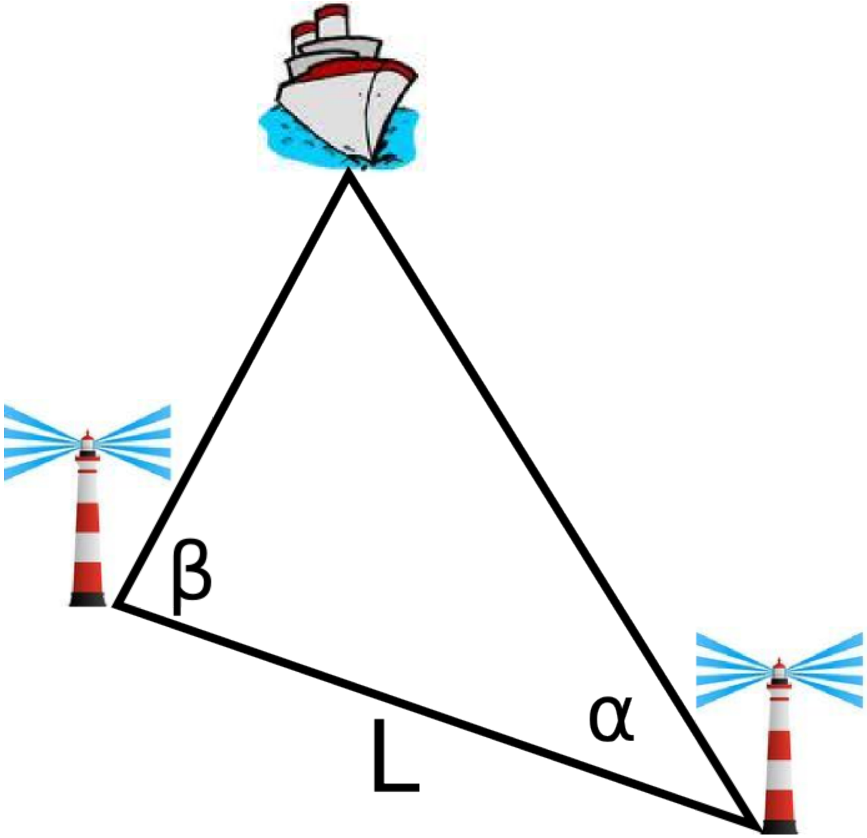
Angle

If we know the (distance) or **angle** to fixed objects on the globe we can also know our position.

Triangulation is the process of determining the location of a point by measuring angles to it from known points at either end of a fixed baseline.

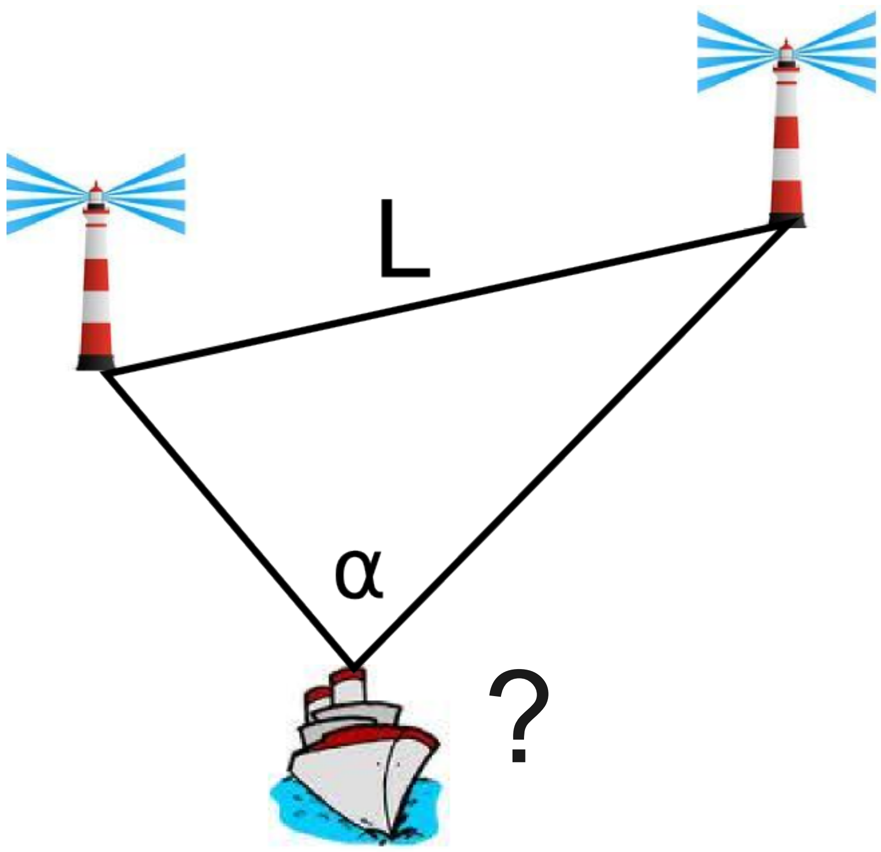
This is based on **trigonometry** (know three things of a triangle and you know everything of the triangle).

For example: *l*, α, β. We know the distance to the ship.



Angle

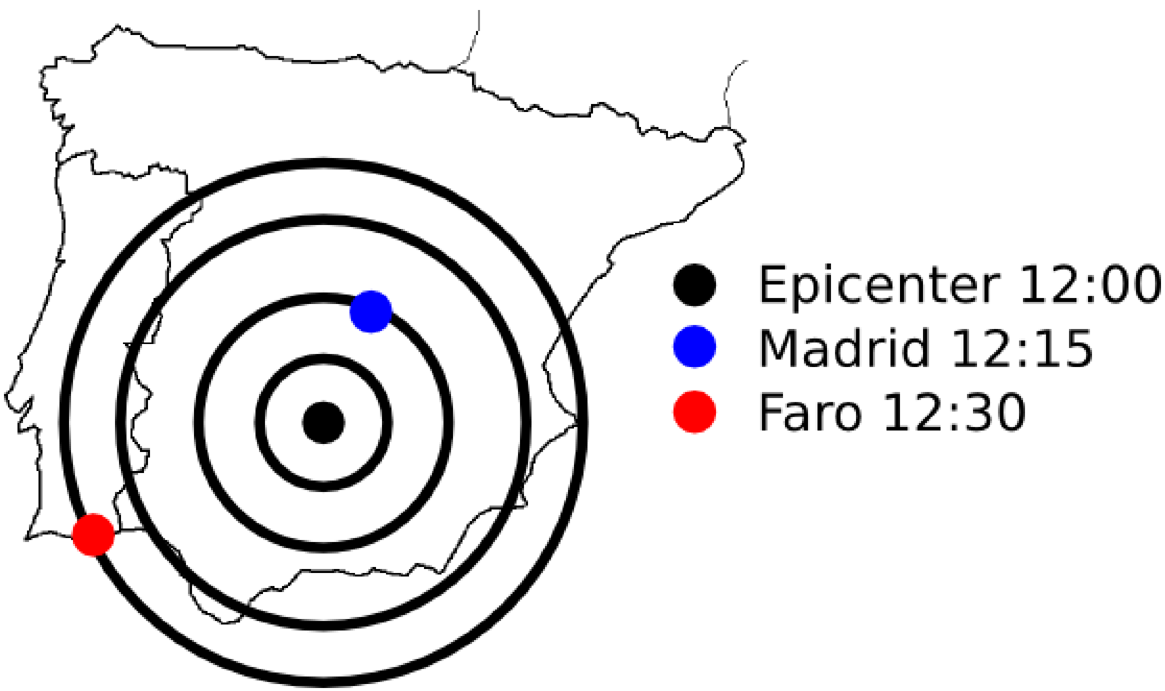
Note that we know where the ship is, but the ships passengers do not know where they are themselves: Lack of information!



Distance

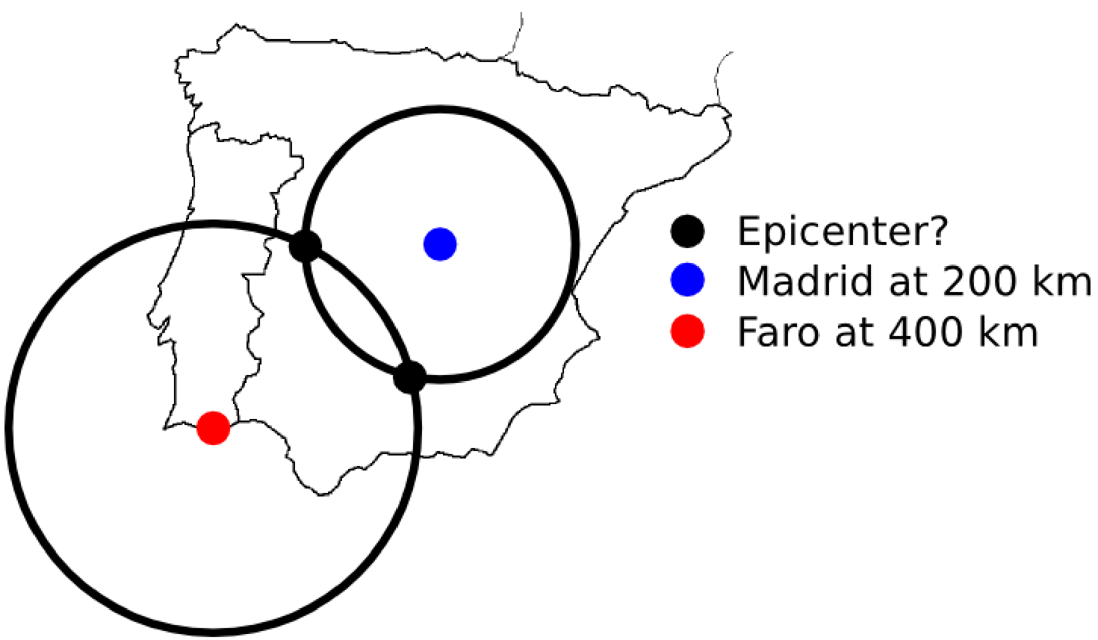
If we know the **distance** (or angle) to fixed objects on the globe we can also know our position.

Example: How is the epicenter of an earthquake measured?



Distance; Earthquake

Knowing that seismic waves travel at 5 km/s, knowing the time of arrival of the waves tells us the distance.
Madrid: 40 s later = 200 km
Faro: 80 s later = 400 km
Two solutions:



Apart from that: We need at least 3 (!) stations to determine the epicenter. We don't know **when** the earthquake happened

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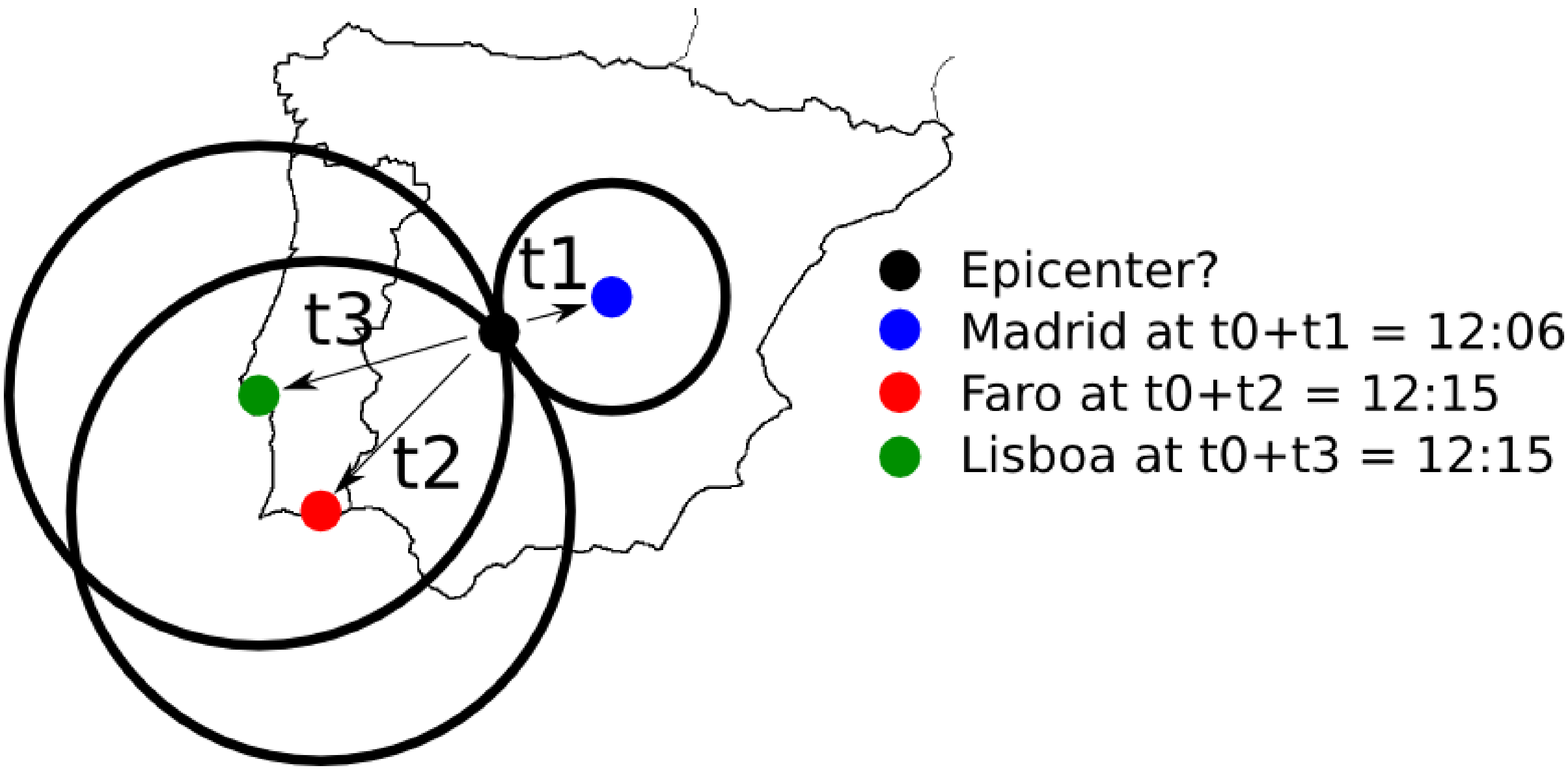
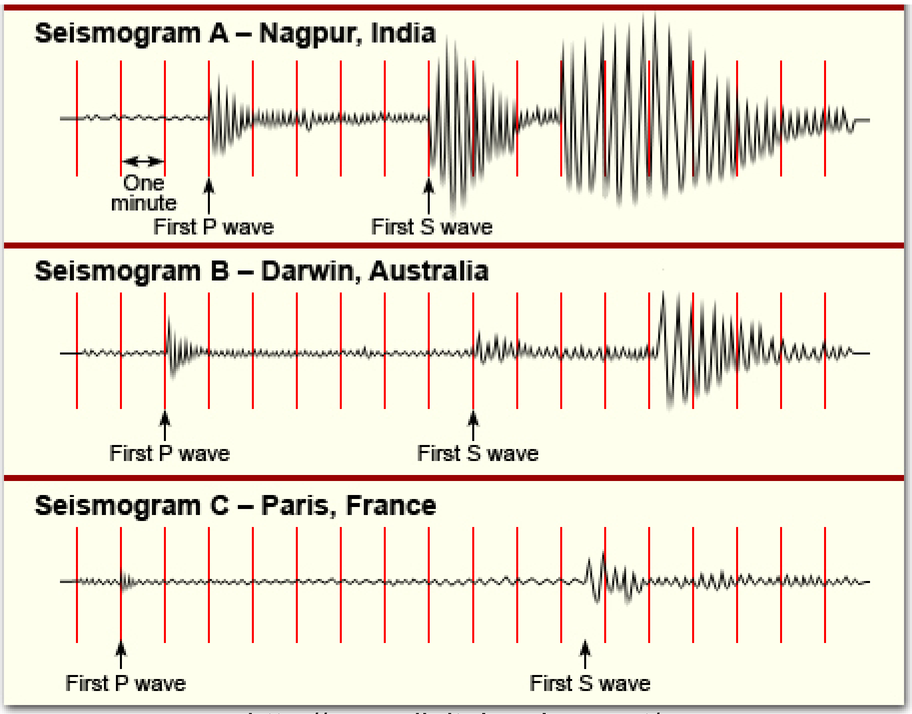


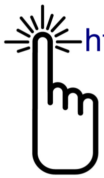
We need at least 3 stations to determine the epicenter. Then we also know **when** the earthquake happened

Distance; Earthquake

In practice difference between arrival of S-waves and P-waves is used (S-waves are transverse, P-waves are longitudinal [like sound waves])



http://www.digitalgeology.net/



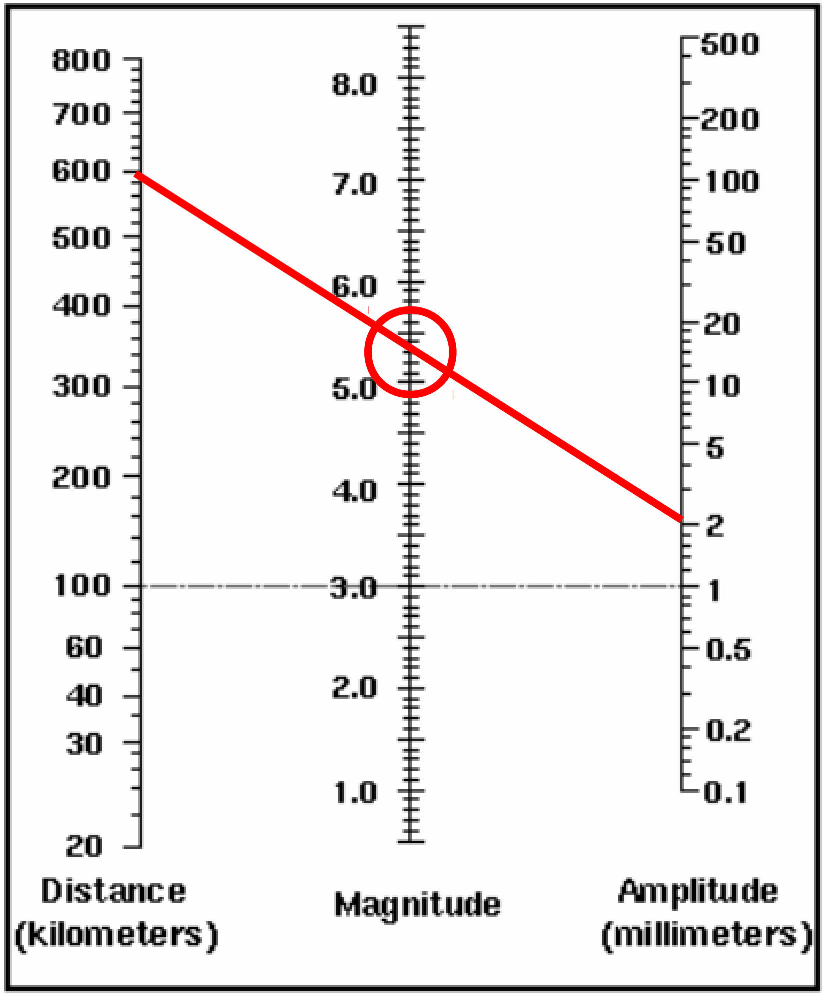
http://www.tasaclips.com/animations/P_S_wave_velocity.html

P-waves: 5 km/s
S-waves: 3 km/s

The difference in arrival of P-waves and S-waves tells us directly the distance

Distance; Earthquake

We can also determine the **magnitude** on the Richter scale



Richter nomogram

Example:
At 600 km, a 5.4 Richter-scale earthquake gives 2 mm amplitude of ground movement

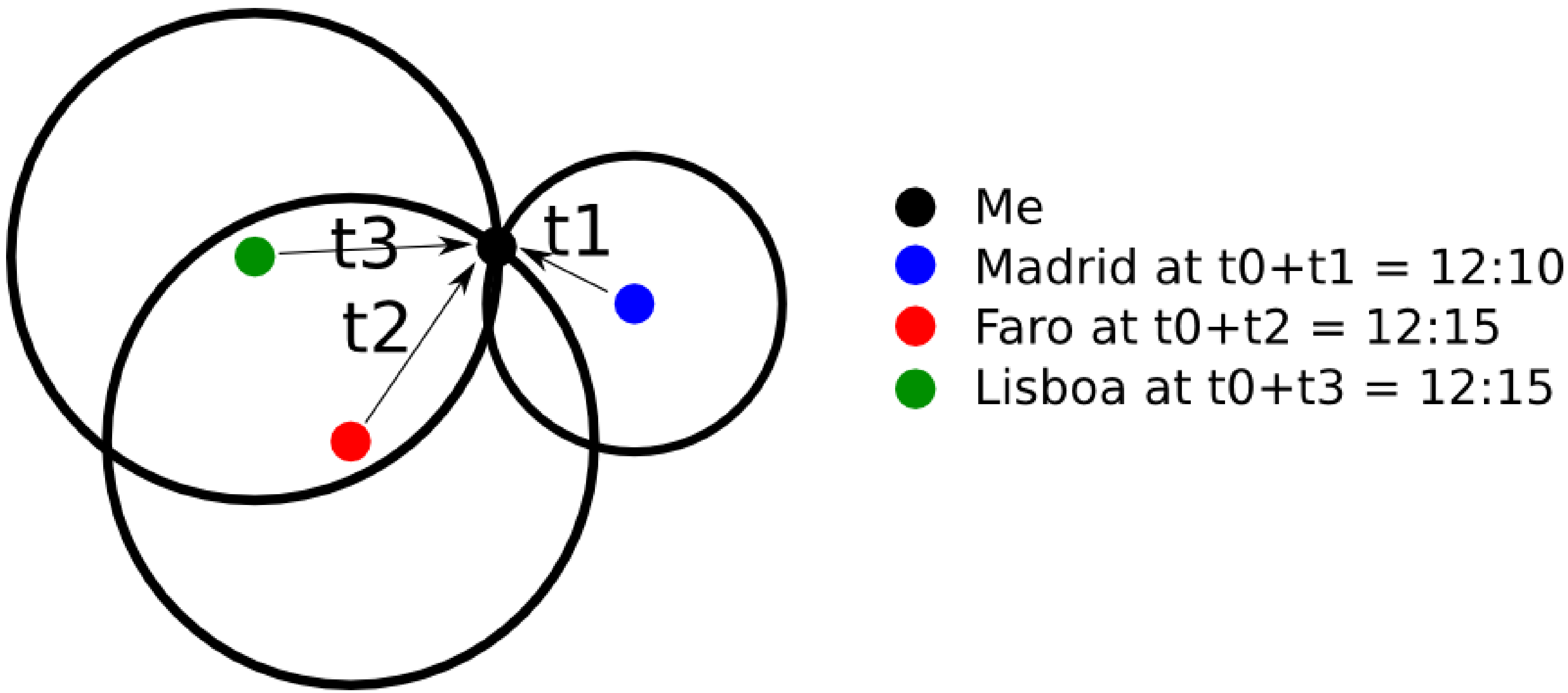
$$A = (1\ \mu m) \times 10^{M} \times (100\ km/D)^{1.86}$$

Logarithmic scale:
A scale-4 earthquake is 10 times stronger than a scale-3 earthquake
Power-Law scale:
10 times further away, $10^{1.86}$ times weaker

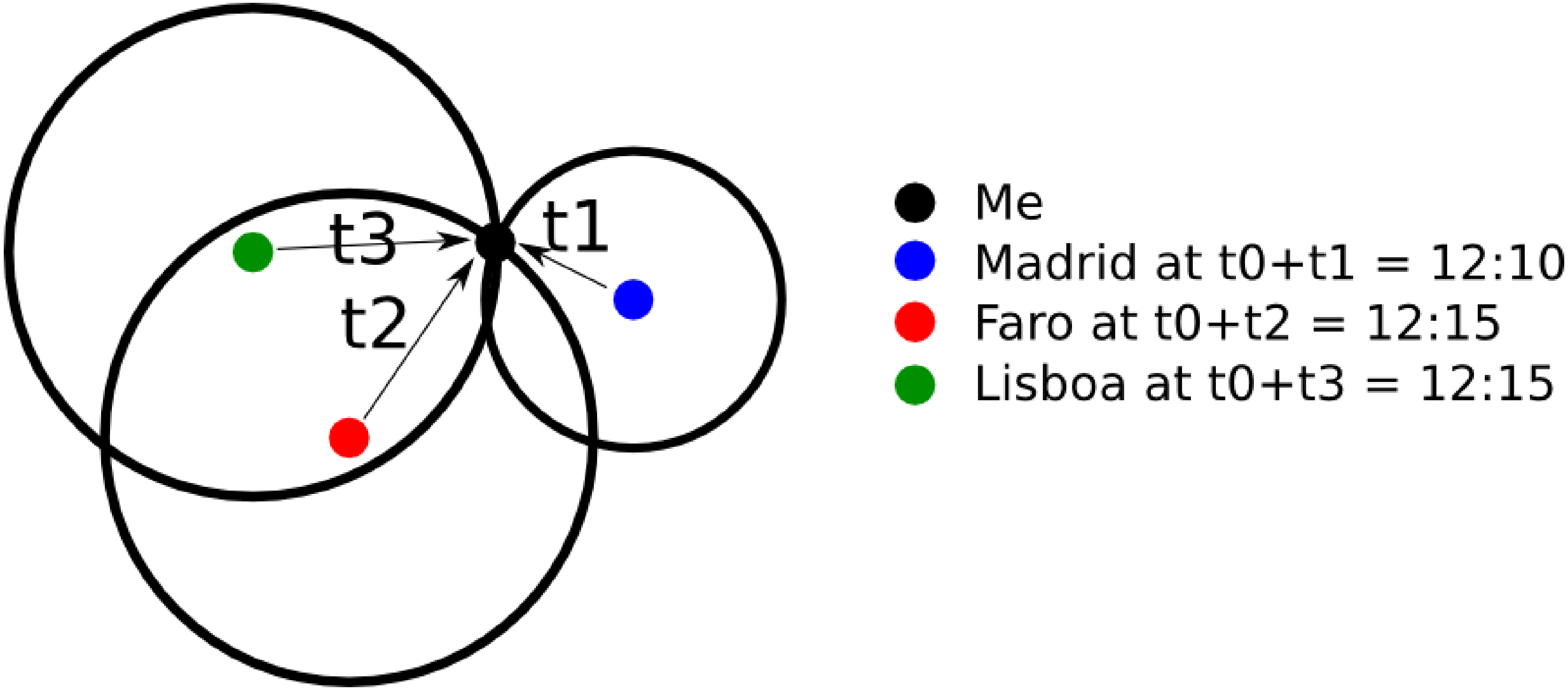
Own Position; inverted problem

Determining our own position is the inverted problem:

Instead of the epicenter firing in all directions and the different location detecting at different times
All locations firing a signal at same time and I (at epicenter) will receive them at different times



Own Position; inverted problem



Positions of stations should be **very** well known

All stations should signal at **exactly** the same time

The arrival time of the signals then tells me the distance to each of the stations and (by trilateration) I can find my own position

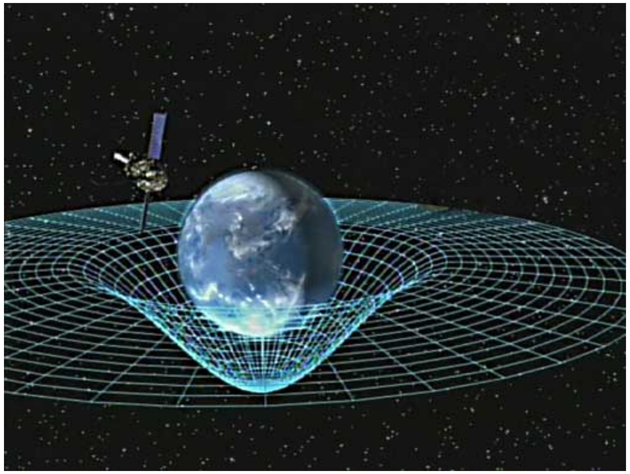
GPS

GPS works with (24) satellites in orbit around the Earth, which allows for determining **longitude**, **latitude** and **height**

Electromagnetic waves traveling with speed of light, $c = 3\text{x}10^8$ m/s

We want position within, say, 10 m.

Timing should be accurate within (10 m)/($3\text{x}10^8$ m/s) = 33 ns.



GPS, satellites with atomic clocks

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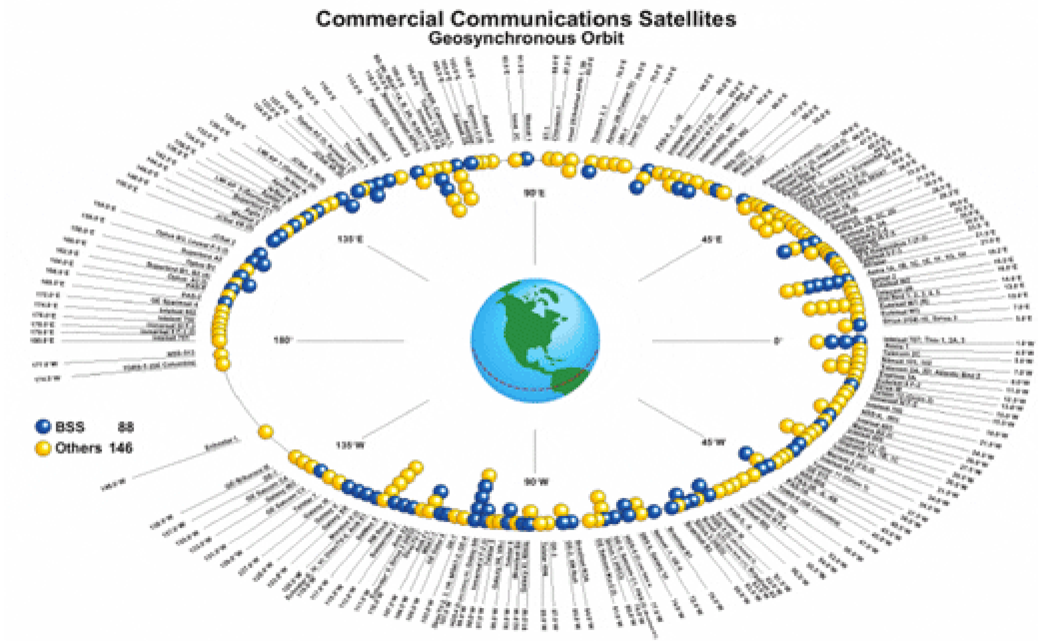
Atomic clocks on all satellites. (No atomic clock needed on GPS receiver!), calibrated constantly by a ground station

Relativistic (Einstein) effects important. (In fact, **GPS is the result of a scientific study on relativistic effects**)

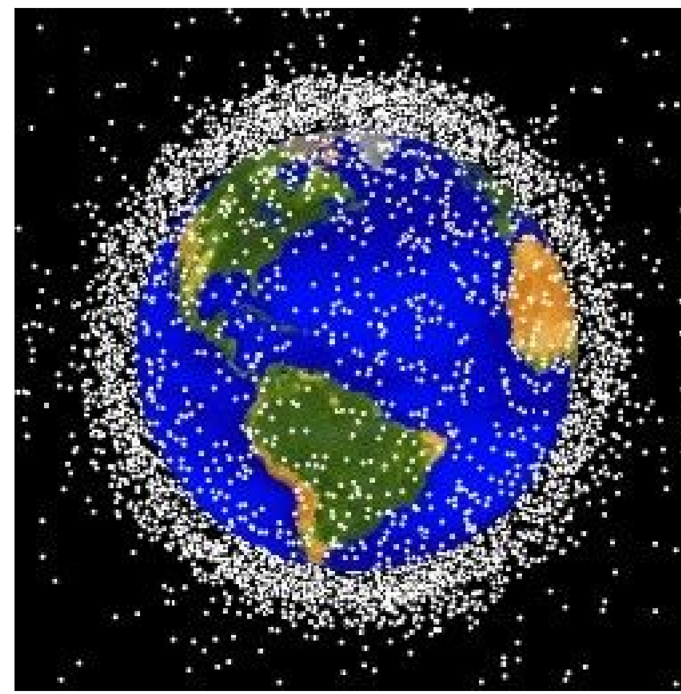
4 satellites are enough, but with more (there are 24 today) we get better accuracy

Satellites in space. Where is Greenpeace?!!

Figure 3: Geostationary Satellites by Orbital Location



Geostationary (communication) satellites



All satellites in Earth space

One way

In GPS, only the satellites send signals. The receiver not.
You do not reveal your location to anybody!

The position of mobile phones can also be determined by looking at the time delays to/from the cellular towers and the signal strength.
They know where you are!

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Whatever you do, **don't** bring your mobile phone!!